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Sound produced by an oscillating arc in a high-pressure gas

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We suggest a simple theory to describe the sound generated by small periodic perturbations of a cylindrical arc in a dense gas. Theoretical analysis was done within the framework of the non-self-consistent channel arc model and supplemented with time-dependent gas dynamic equations. It is shown that an arc with power amplitude oscillations on the order of several percent is a source of sound whose intensity is comparable with external ultrasound sources used in experiments to increase the yield of nanoparticles in the high pressure arc systems for nanoparticle synthesis. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4985805]

I. INTRODUCTION

It is known that a burning arc in a high pressure gas can be a source of high intensity sound. These arcs are known as hissing arcs.[1–3] Usually, the frequency of such sound waves is on the order of hundreds or thousands of Hz. This sound can be related to the injection of erosion jets from electrodes,[4] spatial oscillations of the arc,[5] or with periodic changes of the volt-ampere characteristic. The last one can be stimulated, for example, when an arc is maintained by an AC source or by fluctuations of the current and the voltage in a circuit with a DC source. Thus, the amplitude of power oscillations in an arc discharge experiment with copper electrodes in the atmosphere is about 1%–10%.[6,7] Whereas in an arc discharge experiment with carbon electrodes in a noble gas, there can be intense transverse oscillations of the arc channel with a frequency of about 100 Hz and current amplitude oscillations of up to 50%.[5]

In the peripheral region of an arc with graphite electrodes burning in a high pressure inert gas, a large number of microscopic soot particles are produced together with nanoparticles. Intensive soot generation significantly reduces the efficiency of the arc as the process results in the production of fullerenes and other nanoparticles. Experimental studies have shown that exposure of the peripheral region of the arc to intense ultrasound leads to a noticeable increase in the efficiency of the synthesis of nanoparticles and to the reduction in the yield of soot (see, e.g., Ref. 8). It was shown in Ref. 9 that ultrasound, acting on the suspension of soot microparticles and nanoparticles in an inert gas, results in the coagulation of soot particles, without noticeably affecting the small-scale nanoparticles. For larger particles, the effect is stronger, and soot particles will be brought together in a few seconds or even less. Thereafter, they fall out of the volume under the influence of gravity (similar to a standard industrial method of ultrasonic cleaning of gases). It is shown in this paper that relatively small fluctuations of power in a high-pressure arc can be a source of high-intensity sound comparable to that used in experiments.[8]

We will not consider the near electrode sheaths that can be a source of current and voltage oscillations and therefore a source of power oscillations. Thus, we consider the conventional cylindrical channel arc model[10,11] with an effective channel radius $r = r_{ch}$ (Fig. 1) and a given oscillating source of power (Joule's heat). The self-consistent solution of the channel model will not be found. For the sake of simplicity of analysis, let us assume the given parameters of an equilibrium arc channel not bounded by walls. To obtain the parameters of the generated sound, the arc channel model was supplemented with time-dependent gas dynamic equations.

The parameters of the arc in the channel model are defined by the balance of Joule power

$$Q_0(r) = j^2(r)/\sigma(r), \ \ \mathrm{W/m^3}, \tag{1}$$

and heat losses due to heat conduction.[10] Here, $j(r), \sigma(r)$ are averaged over time local values of current density and conductivity in the channel model. We will consider the arc with a thermal source at equilibrium (1) with small oscillating perturbations.

$$Q(r,t) = Q_0(r) + Q'(r,t), \quad |Q'|/Q_0 \ll 1. \tag{2}$$

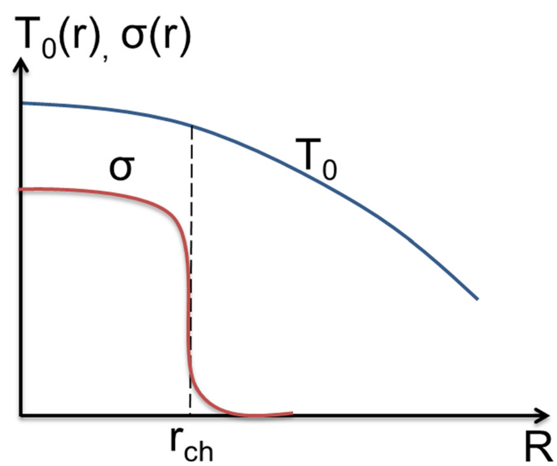

FIG. 1. Qualitative radial distributions of the equilibrium temperature $T_0(r)$ and conductivity $\sigma(r)$ in the channel model.

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The amplitude and the frequency of the perturbations determine the parameters of the radiated cylindrical sound wave.

II. THEORETICAL MODEL AND RESULTS

Let us find the characteristic parameters of a sound wave created by perturbations of the heat source power $Q(r,t)$ in the arc channel. We start with the following standard set of equations[12]

$$\rho T\left(\frac{\partial s}{\partial t}+v\frac{\partial s}{\partial r}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T}{\partial r}\right)+Q(r,t), \tag{3}$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(r(\rho v))=0, \tag{4}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{5}$$

$$p=\rho RT/\mu, \tag{6}$$

where ρ is the density, κ is the thermal conductivity of the air plasma, and v, s, p, μ, and R are the radial velocity, entropy, pressure, averaged molar mass and gas constant of air. Equation (3) expresses the first law of thermodynamics for a given part of the moving gas; Eqs. (4) and (5) represent the conservation of mass and the Euler equation for momentum conservation; Eq. (6) is the equation of state for an ideal gas. While we consider small perturbations of the arc at equilibrium, we can neglect the power loss caused by radiation, because the typical temperature at the axis is about $T\sim 6000$–8000 K.[9,10] For definiteness, we assume that $T(r=0)=7000$ K. At such temperature, ionization in the air plasma is quite small, $\sim 10^{-4}$,[10] thus we can use the equation of state (6) and neglect contributions of the other components of plasma in the equation of state.

As mentioned before, we will consider relatively weak perturbations of power (2) and solve equations (3)–(6) with the use of perturbation theory. The zeroth order solutions have the following form:

$$s=s_0,\ \ T=T_0(r)\ \ p=p_0,\ \ v=0\,,\ \ \rho=\rho_0(r) \\ =p_0\mu/RT_0(r). \tag{7}$$

The only unknown function is $T_0(r)$, which can be found using the channel arc model.[10] As already mentioned in this paper, we do not calculate the equilibrium temperature self-consistently, but we consider it to be given, $T(0)\approx T(r=r_{ch})$. In the cylindrical geometry outside the region of energy release (at $r\geq r_{ch}$), $T_0(r)\approx T(0)-\frac{Q_0 r_{ch}^2}{2\kappa}\ln\left(\frac{r}{r_{ch}}\right)$.

The first order of perturbation theory is given by linearizing (3)–(6)

$$\frac{p_0\mu}{R}\frac{\partial s'}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T'}{\partial r}\right)+Q'(r,t), \tag{8}$$

$$\frac{\partial \rho'}{\partial t}+\frac{\rho_0(r)}{r}\frac{\partial}{\partial r}(rv')=0, \tag{9}$$

$$\frac{\partial v'}{\partial t}=-\frac{1}{\rho_0(r)}\frac{\partial p'}{\partial r}. \tag{10}$$

where the perturbation of every quantity is denoted with a prime. The system of three equations (8)–(10) contains five unknown functions s', T', v', ρ', p'. It is convenient to express all these variations only through p' and T' using the law of thermodynamics (3) and the equation of state (6)

$$\rho'=\mu p'/RT_0-\mu p_0 T'/RT_0^2.$$

From the entropy variation $ds=(\partial s/\partial T)_p dT+(\partial s/\partial p)_T dp$ follows $ds=c_p\left(\frac{dT}{T_0(r)}\right)+\left(\frac{\partial s}{\partial p}\right)_p dp$, where we have used the definition of specific heat c_p.[13] To get the partial derivative of entropy with respect to pressure, we use the standard thermodynamic relations[13]

$$\begin{aligned}(\partial s/\partial p)_T&=-(\partial s/\partial T)_p(\partial T/\partial p)_s\\&=-(\partial s/\partial T)_p(T_0/p_0)(\partial \ln T/\partial \ln p)_s\\&=-(\gamma-1)c_p/\gamma p_0,\end{aligned} \tag{11}$$

where $\gamma=c_p/c_v$. Finally, we get

$$c_p\rho_0(r)\frac{\partial T'}{\partial t}-\frac{\gamma^{\#}\mu c_p}{R}\frac{\partial p'}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T'}{\partial r}\right)+Q'(r,t), \tag{12}$$

$$\frac{\partial p'}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T'}{\partial r}\right)+Q'(r,t), \tag{13}$$

$$\frac{\rho_0(r)}{p_0}\frac{\partial^2 p'}{\partial t^2}-\frac{\rho_0(r)}{T_0(r)}\frac{\partial^2 T'}{\partial t^2}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right)=0, \tag{14}$$

where we have introduced the notation $\gamma^{\#}=1-1/\gamma$. The system of equations (12)–(14) is linear, but has quite a high order, which makes it difficult to solve it analytically. Also, the presence of the non-trivial function of temperature $T_0(r)$ makes the problem more complicated. Nevertheless, we can simplify the given set of equations. Let us estimate whether we can neglect the heat conduction term $\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T'}{\partial r}\right)$ in Eqs. (12) and (13). From Eq. (12), we can deduce the characteristic scale for changing temperature l_T and compare it with the arc channel radius r_{ch}. By order of magnitude

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa\frac{\partial T'}{\partial r}\right)\sim\frac{\kappa\cdot T'}{l_T^2},\ \ \frac{\mu c_p}{R}\omega p'\sim c_p\rho_0(r)\omega T'.$$

Thus, the heat conduction term can be neglected, if

$$l_T^2\gg\frac{\kappa}{2\pi c_p\rho_0(r)f}. \tag{15}$$

Taking into account that the characteristic thermal scale is $l_T\sim r_{ch}$, from (15), we can obtain an estimate of the range of perturbation frequencies at which the thermal conductivity term can be neglected

$$f\gg\frac{\kappa}{2\pi c_p\rho_0(r_{ch})r_{ch}^2}\sim 600\,\text{Hz}, \tag{16}$$

where we assumed the channel radius $r_{ch}=0.25$ cm and have used the equations of state to show that $\frac{p'}{p}\approx\frac{T'}{T}$, therefore, both terms on the left-hand side of Eq. (12) have the same order. As the usual sound frequency in an experiment is $f \geq 10^3$ Hz, we can neglect the term $\kappa \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right)$. Physically, this means that the temperature perturbation cannot change because of the thermal conductivity during the period of acoustic oscillations. The only physical quantity that propagates fast across the system is pressure. We came to the following set of equations:

$$c_p \rho_0(r) \frac{\partial T'}{\partial t} - \gamma^{\#} \mu c_p R \frac{\partial p'}{\partial t} = Q'(r,t), \tag{17}$$

$$\frac{\rho_0(r)}{p_0}\frac{\partial^2 p'}{\partial t^2} - \frac{\rho_0(r)}{T_0(r)}\frac{\partial^2 T'}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right) = 0. \tag{18}$$

Taking a time derivative of the first equation and substituting it into the second equation, we will get an analog of the wave equation

$$\frac{1}{c_s^2(r)}\frac{\partial^2 p'}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right) = \frac{1}{c_p T_0(r)}\frac{\partial Q'(r,t)}{\partial t}, \tag{19}$$

where $c_s(r) = \left(\frac{\gamma R T_0(r)}{\mu}\right)^{1/2}$ is the corresponding local sound velocity. Because the perturbations of power $Q'(r,t) \neq 0$ exist only inside the arc channel and at the frequency $f = \omega/2\pi \sim 10^3$ Hz, the wavelength of a sonic wave is $\lambda = c_s(T)/f \sim 0.5$ m, so $\lambda \gg r_{ch}$; we may consider that p' does not depend on spatial coordinates inside the arc channel and its vicinity. Equation (19) leads to a relation between the amplitude of the heat released and the pressure perturbations

$$p'_a(r_c) = \frac{c_s^2 Q'}{c_p T_0 \omega} \approx \frac{2}{3}\frac{Q'}{\omega}, \tag{20}$$

where $c_p = 5R/2$, and we have assumed $Q', p'(r,t) \propto e^{-i\omega t}$. Let us introduce the factor $\eta = Q'/Q_0$

$$Q' = \eta Q_0 = \eta \sigma E^2. \tag{21}$$

Outside of the channel, there is no heat production and Eq. (19) is reduced to a standard free wave equation for $p'(r,t)$

$$\frac{1}{c_s^2(r)}\frac{\partial^2 p'}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right) = 0, \tag{22}$$

with the amplitude at the channel boundary given by Eq. (20).

By substituting $p'(r,t) = p'_a(r)e^{-i\omega t}$ into Eq. (22), we get the equation for the amplitude of pressure

$$\frac{\partial^2 p'_a}{\partial r^2} + \frac{1}{r}\frac{\partial p'_a}{\partial r} + \frac{\omega^2}{c_s^2(r)} p'_a = 0. \tag{23}$$

Replacing $p'_a(r)$ by the anzats $p'_a(r) = r^{-1/2} f(r)$ in Eq. (23), we get the following equation on $f(r)$:

$$\frac{\partial^2 f}{\partial r^2} + \left(\frac{1}{4r^2} + \frac{\omega^2}{c_s^2(r)}\right) f = 0. \tag{24}$$

When we consider the solution in the vicinity of the channel $r \sim r_{ch} \ll c_c(r_{ch})/\omega$, we can neglect $\omega/c_s(r_{ch})$ by comparing to $1/r$ and get the solution as

$$f(r) = B r^{1/2} \ln(r/A), \tag{25}$$

where A and B are some constants to be determined. These constants are obtained by considering the region, where $r \sim c_s(r_{ch})/\omega = 1/k_0$. The solution in this region can be approximated as

$$f(r) = r^{1/2}\left[C H_0^1(k_0 r) + D H_0^2(k_0 r)\right]. \tag{26}$$

Expanding this approximation in the vicinity of the channel, we get

$$f(r) \approx r^{1/2}(C + D)\ln(k_0 r). \tag{27}$$

Comparing (25) and (27), we can conclude that $A = 1/k_0 = c_s(r_{ch})/\omega$. Using the boundary condition (20), we can find the constant B in (25) and derive the following relation for the amplitude of pressure:

$$p'_a(r) \approx \frac{2}{3}\frac{Q' \ln(\omega r / c_s(r))}{\omega \ln(\omega r_{ch}/c_s(r_{ch}))}, \tag{28}$$

in the near field region, when $\omega^2 r_{ch}^2 / c_s^2(r_{ch}) \ll 1$ and $\lambda \gg r_{ch}$.

The relation between the pressure and the acoustic oscillatory velocity amplitude $u(r)$ of the wave follows from Eq. (10)

$$\frac{\omega \rho_\infty T_\infty}{T_0(r)} u = \frac{2Q'}{3\omega r \ln(\omega r_{ch}/c_s(r_{ch}))}, \tag{29}$$

where $\rho_0(r) = \rho_\infty T_\infty / T_0(r)$ at constant pressure; ρ_∞ and T_∞ correspond to the density and the temperature of the unperturbed gas at infinity, ($r \gg r_{ch}$). Finally, it gives us the following relation for the amplitude of the oscillatory velocity in the sound wave on the arc channel boundary:

$$u(r_{ch}) \approx -\frac{2Q' T_0(r_{ch})}{3\rho_\infty T_\infty \omega^2 r_{ch} \ln(\omega r_{ch}/c_s(r_{ch}))}. \tag{30}$$

III. DISCUSSION

The values of the pressure and the velocity amplitudes of acoustic oscillations (28) and (30) at the arc channel boundary $r = r_{ch}$ can serve as boundary conditions in the classical problem of an infinite cylindrical sound source in a gas.[12] Since the wavelength of the excited sound in the frequency range of interest to us $\lambda \geq 10$ cm much exceeds the typical sizes of the inter-electrode gap of high pressure arcs, the radiated sound waves degenerate into spherical ones at a distance $\sim\lambda$. However, in the near zone, at a distance of up to several centimeters from the arc channel, the cylindrical approximation is quite valid. In this region, where the gas temperature falls to ~1500 K and below, carbon nanoparticles are synthesized in arc discharges with graphite electrodes in inert high-pressure gases.[14]

An estimate of the sound wave intensity in the near field follows from (28). In conventional units, the intensity is[15]

$$L_p(r) = 20\lg(p'_{rms}(r)/p_{ref}), \tag{31}$$

where $p'_{rms}(r) = p'(r)/\sqrt{2}$ and $p_{ref} = 2 \times 10^{-5}$ Pa is the standard reference sound pressure.

As an example, for the arc in air at $p_\infty = 760$ Torr, $T_\infty = 300$ K, the arc channel radius $r_{ch} = 0.5$ cm, the core temperature $T_0(0) \approx T_0(r_{ch}) = 7000$ K, and oscillation frequency $f = 2000$ Hz, taking into account the typical magnitude of the electric field $E \sim 1000$ V/m and the equilibrium conductivity $\sigma(T(r_{ch})) \approx 50\ \Omega^{-1}\,\mathrm{m}^{-1}$,[10] we get: if $\eta = 0.01$, at $r = r_{ch} = 0.25$ cm and outside the channel at $r_1 = 1.5$ cm, the intensity of the sound wave $L_p(r)$ equals 119.43 and 110 dB, correspondingly. While for $\eta = 0.05$, at the same r_{ch}, r_1, the corresponding values for the ultrasound intensities are 133.4 and 124.4 dB. Thus, we see that the power perturbations in the arc channel, even for amplitudes about a few percent out of averaged Joule power, can produce a high intensity sound, which is of the order or exceeds 100–120 dB in the vicinity of the arc. Such intensity is at the same scale as the sound generated by the external source in the experiments mentioned above.[8]

IV. CONCLUSIONS

We have considered a simple model of sound generation by an electric arc with a fluctuating power source, and estimation formulas for the generated sound intensity were obtained. Small perturbations of Joule heating in a stationary high pressure arc can be an intensive source of sound and, therefore, can have a strong influence on the coagulation of soot and lead to the increased production of fullerenes and nanoparticles.

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